MATH 2551, Fall 2018
Practice Final

Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One sheet of paper (letter size) for formulas (one side) is allowed. Calculator is not allowed in this exam. Try to work this practice within 120 minutes.

Problem 1. This problem is about the function

$$f(x, y, z) = 3zy + 4xcos(z).$$

(a) Find the rate of change of the function f at $(1, 1, 0)$ in the direction from this point to the origin.

(b) Give an approximate value of $f(0.9, 1.2, 0.11)$

(c) The equation $f(x, y, z) = 4$ implicitly defines z as a function of (x, y), if we agree that $z = 0$ if $(x, y) = (1, 1)$. Find the numnerical values of the derivatives:

$\frac{\partial z}{\partial x}(1, 1)$ and $\frac{\partial z}{\partial y}(1, 1)$.

(d) Suppose $\mathbf{r}(t) = (x(t), y(t), z(t))$ is a parametric curve such that $\mathbf{r}(0) =$

$(1,1,0)$ and $\mathbf{r}'(0) = (3,2,1)$. Find the value of

$$\frac{d}{dt} f(\mathbf{r}(t))|_{t=0}.$$

Problem 2. Consider the planar vector field

$\mathbf{F}(x,y) = (y-2x^2)\mathbf{i}+4\mathbf{j}$, and $\mathbf{G}(x,y) = (1+xy)e^{xy}\mathbf{i}+x^2e^{xy}\mathbf{j}$, and the curve C from point $A(-2,0)$ to $B(1,3)$ that goes along the parabola $y = 4 - x^2$.

(a) Is $\mathbf{F}$ a gradient field? If yes, find a function whose gradient is $\mathbf{F}$.

(b) Is $\mathbf{G}$ a gradient field? If yes, find a function whose gradient is $\mathbf{G}$.

(c) Compute the work done by the field $\mathbf{F}$ along the curve C.

(d) Compute the work done by the field $\mathbf{G}$ along the curve C.

Problem 3. Evaluate $I = \int_{C_R} dx + x^2y dy$, where C_R is the triangle with vertices $(0,0)$, $(0,R)$, $(R,0)$ oriented counterclockwise.

Problem 4 Let S be the portion of the surface $x = 5 - y^2 - z^2$ in the half space $x \geq 1$, oriented so that the normal vector at $(5,0,0)$ is equal to $\mathbf{i}$. Let $\mathbf{F}(x,y,z) = -\mathbf{i} + \mathbf{j}$ (a constant vector field).

(a) Set up and evaluate the flux of $\mathbf{F}$ across S.

(b) Verify that $\mathbf{F} = \nabla \times \mathbf{G}$, where $\mathbf{G} = z\mathbf{j} - x\mathbf{k}$.

(c) Give an alternative calculation of the surface integral of part (a) by applying Stokes' theorem.

Problem 5 Find and classify all critical points of the function

$$f(x, y) = \frac{5}{2}x^2 - xy + 15x + \frac{1}{75}y^3 - 3y$$

Problem 6 True or False? Circle the correct answer. No partial credit.

- 1 : Any constant vector field $\mathbf{F}$ is a gradient field.

  (a) True (b) False.

- 2: If C_1 and C_2 are two oriented curves, $\mathbf{F}$ is a vector field, and the length of C_1 is greater than the length of C_2, then $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} > \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.

  (a) True (b) False.

- 3: If $\nabla \cdot \mathbf{F} = 0$ and $\nabla \times \mathbf{F} = \mathbf{0}$, then $\mathbf{F} = \mathbf{0}$.

  (a) True (b) False.

- 4: If S_1 and S_2 are two oriented surface bounded by the same positively oriented curve C and $\mathbf{F}$ is a smooth vector field, the the flux of $\nabla \times \mathbf{F}$ through S_1 and S_2 are the same.

(a) True (b) False.

- 5 : If S is a unit sphere centered at the origin and $\mathbf{F}$ is a vector field that has zero total flux out of S, then $\nabla \cdot \mathbf{F} = 0$ at all points inside S.

  (a) True (b) False.

Problem 7 Consider the surface S that is given by the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 3$.

(a) Give a parametirc representation of S. Make sure to explicitly describe or sketch the parametrization domain D.

(b) Find an equation of the tangent plane to S at the point $P(-1, 1, \sqrt{2})$.

(c) If the density function $\lambda(x, y, z)$ is equal to the distance to the xy-plane, find the total mass of the surface S.

Problem 8 Let E denote the portion of the solid ball of radius R centered at the origin in the first octant, and let

$$\mathbf{F} = (2x + y)\mathbf{i} + y^2\mathbf{j} + cos(xy)\mathbf{k}.$$

Applying the Divergence Theorem, compute the net flux of the field $\mathbf{F}$ across the boundary of E, oriented by the outward-pointing normal vectors.

Problem 9 Please complete the course survey. Your comments will help me to improve my teaching in the future. Thank you in advance.