

**MATH 4305, Fall 2016,
Midterm 2, Practice**

Show all your work. You may use one side of a letter-size sheet of paper for formulae in this exam. Calculator is not allowed. Please give yourself 50 minutes.

Problem 1 Let $\mathbf{y} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, and $W = \text{span}\{\mathbf{u}, \mathbf{v}\}$.

a) Find the orthogonal projection of $\mathbf{y}$ onto W .

b) Find the distance between $\mathbf{y}$ and W .

Problem 2 Find the trigonometric function of the form $f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t)$ that best fits the data points $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$, using least squares. Compute the least square error. (Remark: This is a problem for concept, find the formula, don't have to solve for exact solution. The test problem will be easier to solve.)

Problem 3 Find all possible values of a so that the columns of A given below are linearly dependent?

$$\begin{pmatrix} a & 2a & 0 & 0 \\ 0 & 0 & a-3 & 3(a-3) \\ 0 & -2a & 0 & 1 \\ 0 & 0 & a-2 & 2(a-2) \end{pmatrix}$$

Problem 4 (a) Prove that the set $\mathbf{B} = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for $\mathbf{P}_2$.

b) Find the matrix of the linear transformation $T(f(t)) = f' - 3f$ from $\mathbf{P}_2$ to $\mathbf{P}_2$ with respect to the basis $\mathbf{B}$ found in part (a).

Problem 5. Let A be the following matrix

$$\begin{pmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix}$$

a) Find the QR factorization of A .

b) Find the orthogonal projection of $\mathbf{b} = (1, 2, 3, 4)^T$ onto $\text{Col}(A)$.

Problem 6: If A is an $n \times n$ matrix, is it true that $\det(AA^T) = \det(A^T A)$?
Why?