Guideline: Please read the following carefully.

Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. One side of a sheet of paper for formulas. Calculator is not allowed in this exam. You have 50 minutes.

Problem 1. Calculations.

(a) Find the directional derivative of \( f(x, y, z) = xy + yz + zx \) at \( P(1, -1, 1) \) in the direction of \( i + 2j + k \)

(b) Find the rate of change of \( f(x, y) = xe^y + ye^{-x} \) along the curve \( r(t) = (\ln t)i + t(\ln t)j \).

(c) Find \( \frac{\partial u}{\partial s} \) for \( u = x^2 - xy, x = scost, y = tsins \).

(d) Find \( \frac{du}{dx} \) if \( xcos(xy) + ycos(x) = 2 \).

(e) Is \( F(x, y) = (x + siny)i + (xcosy - 2y)j \) a gradient of a function \( f(x, y) \)? If yes, find the general form of \( f(x, y) \).

(f) Set \( f(x, y) = \frac{x^2 - y^4}{x^3 - y^4} \). Determine whether or not \( f \) has a limit at \((1,1)\).

Problem 2 Consider the function \( f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z} \).
(a) Find the equation for the tangent plane to the level surface $f = 4$ at the point $P(1, 4, 1)$.

(b) Find the equation for the normal line to $f = 4$ at $P(1, 4, 1)$.

(c) Use differentials to estimate $f(0.9, 4.1, 1.1)$.

Problem 3. Find the area of the largest rectangle with edges parallel to the coordinate axes that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Problem 4 Find the absolute extreme values taken on $f(x, y) = \frac{-2y}{x^2+y^2+1}$ on the set $D = \{(x, y) : x^2 + y^2 \leq 4\}$.