Show all your work. You may use one side of a letter-size sheet of paper for formulae in this exam. Calculator is not allowed. Please give yourself 50 minutes.

**Problem 1**

Let \( y = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \), \( u = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \), \( v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \), and \( W = \text{span}\{u, v\} \).

a) Find the orthogonal projection of \( y \) onto \( W \).

b) Find the distance between \( y \) and \( W \).

**Problem 2**

Find the trigonometric function of the form \( f(t) = c_0 + c_1 \sin(t) + c_2 \cos(t) \) that best fits the data points \((0, 0), (1, 1), (2, 2), (3, 3)\), using least squares. Compute the least square error. (Remark: This is a problem for concept, find the formula, don’t have to solve for exact solution. The test problem will be easier to solve.)

**Problem 3**

Find all possible values of \( a \) so that the columns of \( A \) given below are linearly dependent?

\[
\begin{pmatrix}
  a & 2a & 0 & 0 \\
  0 & 0 & a - 3 & 3(a - 3) \\
  0 & -2a & 0 & 1 \\
  0 & 0 & a - 2 & 2(a - 2)
\end{pmatrix}
\]

**Problem 4**

(a) Prove that the set \( B = \{1 + t^2, t + t^2, 1 + 2t + t^2\} \) is a basis for \( P_2 \).

b) Find the matrix of the linear transformation \( T(f(t)) = f' - 3f \) from \( P_2 \) to \( P_2 \) with respect to the basis \( B \) found in part (a).

**Problem 5**

Let \( A \) be the following matrix

\[
\begin{pmatrix}
  1 & 3 & 5 \\
  1 & 1 & 0 \\
  1 & 1 & 2 \\
  1 & 3 & 3
\end{pmatrix}
\]

a) Find the \( QR \) factorization of \( A \).

b) Find the orthogonal projection of \( b = (1, 2, 3, 4)^T \) onto \( \text{Col}(A) \).
Problem 6: If $A$ is an $n \times n$ matrix, is it true that $\det(AA^T) = \det(A^TA)$? Why?